

Computing Shape Similarity with Chaotic Reaction Diffusion Spectra

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Abstract

A novel approach to shape recognition has been developed using a hybrid system of nonlinear reaction diffusion and statistical matching. The reaction diffusion process is implemented using a chaotic coupled map lattice initialized with states derived from a binary image of the shape. The initial set of states from the set $\{0,1\}$ is transformed by the dynamics into a rich distribution of states in the range $[-1, 1]$. Histogram binning of the state distribution is used as the basis for statistical matching. Results on a set of test shapes compare favorably with human judgments of shape similarity, and may assist in the search for a biologically grounded theory of shape perception. The paper describes the method, experimental results, and introduces some of the theoretical background motivating the approach.

Keywords: shape similarity, shape recognition, coupled map lattices, cellular neural networks, chaotic neural networks, applications of chaos, reaction-diffusion, image databases, content-based retrieval

1. Introduction

Successful categorization of shapes or identifying a shape most similar to a prototype are fundamental tasks in computer vision, and serve as a basis for higher levels of object and scene recognition in images and for object tracking in video sequences. Research into the problem is increasingly motivated by new applications in content-based retrieval for image and video databases. In multimedia applications it is particularly important to conform to user judgments of similarity, and to have a representation which can be matched with efficient and, ideally, with parallel computations. A few recent studies have attempted to compare human similarity judgments with the results of testing several different algorithms. Mumford et. al. [Mumford 89] studied the confusability of fifteen simple equal perimeter polygons for pigeons and for a group of twenty-eight human subjects. The best performing algorithm (feature graph matching) achieved a correlation of .8 with the human data after an ad hoc fix to emulate human confusion of mirror images.

The diffusion spectrum method described here was tested on the Mumford polygons and was found to agree with the human best match in 9 of 15 shapes, and to include the human best match in the top 3 matches in 11 of 15. The system for shape categorization and recognition described here is essentially a hybrid. A chaotic map lattice front end generates patterns, which are ultimately treated statistically as populations. No training is necessary; categorization is intrinsic in the network dynamics. The approach promises to address one of the early criticisms of theories on the role of chaos in biological perception [Skarda 87], showing that chaotic reaction diffusion, particularly with non-stationary dynamics, may indeed be useful in computing similarity of signals.

2. Diffusion Spectra Method and System

Reaction-diffusion models have been studied for over 40 years to describe pattern formation processes, and are currently in use for image processing applications such as halftoning [Sherstinsky 94]. Until recently these problems have been formulated as partial differential equations (PDE), and solved with computationally expensive numerical methods [Price 87, 93]. An alternative formulation for such systems known as coupled map lattices (CML) based on discrete time, discrete space, and real-valued state representations of complex spatially coupled systems was introduced by Kaneko [Kaneko 89, 93]. Similar models have appeared more recently in the neural network literature described as cellular neural networks [Cimagalli 93], and earlier as fractal chaos networks [Perez 87] and cellular dynamata [Abraham 91]. These models are related to the branch of neural network analysis involving dynamic patterns in cell assemblies [Katchalsky 74], [Wilson 72], and most visible today in study of perception in the olfactory bulb and the encoding of perceptual stimuli as chaotic attractors in a spatially distributed network [Yao 91].

The notion that discrete temporal dynamics may play a large role in perception and learning is a relatively new one. Discrete time clocking dynamics may be used in cortical assemblies at different time scales to establish (fast) entrainment and (slower) bifurcation frame rates; at frame boundaries phase dependent Hebbian learning can occur [Baird 90]. The characteristic oscillation rates seen in cortex that might correspond to the iterations in coupled cell assemblies produce a reasonable fit to the response time data in human shape recognition experiments, using the numbers of iterations found to be effective so far. Specifically, 40 Hz signals are identified as likely candidates for entrainment cycles; while recognition times reported for the shapes here ranged from 400-550 ms. This puts an upper bound on the number of iterations to compute and match against a target shape of 16-22 iterations, if computations similar to those considered here are involved in shape discrimination. 4-8 iterations have been used in these experiments, allowing the remaining time to be used for comparison or memory activation operations.

A coupled map lattice (CML) is a dynamical system with discrete time, discrete space, and real valued state. The lattice consists of field variables representing macroscopic (distributed) qualities, such as temperature, fluid velocity field, local concentration of a chemical substance, or in our case neuron pulse density in a local assembly with diffusion (entrainment) to neighboring assemblies. The process is decomposed into simple parallel dynamics at each lattice point, with each process carried out successively.. In the present model, this means that at each iteration, a diffusion step is performed, then a reaction step.

The entire diffusion step can be expressed as:

$$S_d(x, y) = (1 - c)S_t(x, y) + \frac{c}{4}[S_t(x, y + 1) + S_t(x, y - 1) + S_t(x + 1, y) + S_t(x - 1, y)]$$

where S_d is the intermediate diffusion array, t is the current time step, x, y are the spatial indices of the pixel array S at the center of the diffusion neighborhood, S is the state variable at each pixel of the array, and c is the coupling constant restricted to the range [0.0 to 1.0].

The second computational unit applied in each time step is the logistic map:

$$S_{t+1}(x, y) = 1 - bS_d(x, y)^2$$

where $S, t, x,$ and y are as above and where b is the bifurcation parameter, restricted to the range ($0.0 < b < 2.0$). S is restricted to the range ($-1.0 < S < 1.0$).

The essence of the method here is to transform the original image, assumed to be a binary intensity image, into a richer distribution of intensity values. The distribution evolves through an iterated diffusion process combined with a nonlinear function computed from the current pixel value. After a few iterations the state of the evolved array is a pattern which is characterized by a histogram of intensity values. While information is lost by ignoring spatial relationships in the resulting pattern, the nature of the nonlinear diffusion process results in a distribution which captures many aspects of the original spatial relationships. Angles, slopes of lines and local curvature result in unique local state distributions after a

few time steps. The raw histograms for objects are normalized to reduce any explicit size dependencies, by simply dividing the integer bin counts by the total number of pixels in the object. Shape similarity between any two shapes can then be computed by a simple Euclidean distance metric. Experiments were performed with 64 and 128 bins, where the bin values range from -1.0 to 1.0. It was determined that 128 bins gave better results due to the loss of resolution and to a fractal clustering of the distribution.. Early studies on the period doubling bifurcation points of a single parabolic map show that the attractors in successive bifurcations are clustered in this way [Hofstadter 85], but it came as something of a surprise to the author that a chaotic spatial process would respond this way. The analysis in terms of extreme value theory described in [Miller 92] may be the best explanation for the process, with the coupling and bifurcation parameters placing the dynamics in a critical regime to generate fractal attractor basin distributions.

3. Experimental Results

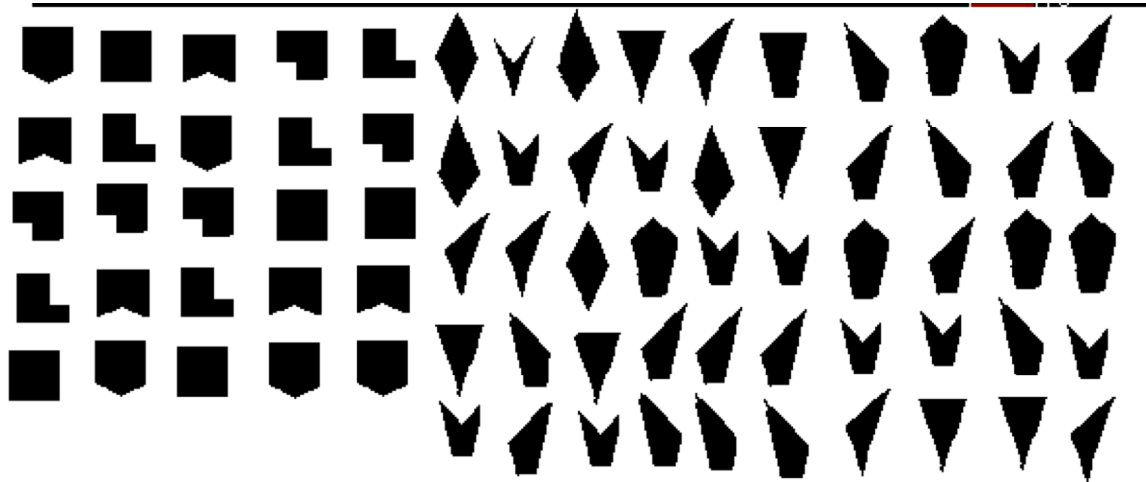


Figure 1: The top row is the target shape; each column beneath a target are the 4 best matches, obtained using 4 iterations at $b=1.5054$, $c=.3317$ followed by 4 iterations at $b=.9055$, $c=.3317$

The parameters used in this experiment could be described as a quenching cycle; the initial bifurcation value $b=1.5054$ is in a weakly chaotic area of the dynamics; running for 4 steps generates a rich distribution of states. Previous experiments with higher b (more chaotic) and more iterations indicate that all shapes tend toward a flatter, noisy distribution. This experiment approaches a noisy distribution, but then moves the maps into a period 2 range ($b=.9055$). After about 20 iterations, the state distribution will collapse into a binary distribution again; in a few iterations in the period two regime some states progress toward the attractor faster than others, and this seems to correlate with the original pattern which generated the attractor. The quenching phase improves performance over a simple 4 step evolution for this parameter set, but it may be unnecessary if better base bifurcation and coupling points were found.

4. Summary

A novel method for computing shape similarity has been developed . Initial tests on a small set of shapes indicate that the method compares favorably with other algorithms in correlating with human similarity judgments; testing on a large set of shapes is underway. A few explorations on imagery with finer feature grain and higher spatial frequencies (characters and fonts) indicate that a multi-scale implementation is probably necessary to handle more diverse recognition tasks. It has been shown that earlier criticism of chaotic neural models being unsuitable for computing similarity measures may not hold when non-stationary dynamics and interacting time scales, such as the quenching cycles reported here, are employed.

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